Higher Maths - Leckie & Leckie

1A

1 (a) $y = e^{3}$

$= 20.0855...$

$= 20.1$ (3 s.f.)

(b) $y = e^{-3}$

$= 0.049787...$

$= 0.0498$ (3 s.f.)

(c) $y = 2e^{0.5}$

$= 3.29744...$

$= 3.30$ (3 s.f.)

(d) $y = e^{-0.2 \times 53}$

$= 2.49160... \times 10^{-5}$

$= 2.49 \times 10^{-5}$

2 (a)(i) $P_1 = 100e^{0.132 \times 1}$

$= 114.11...$

$= 114$ mice

(ii) $P_5 = 100e^{0.132 \times 5}$

$= 193.479...$

$= 193$ mice

(b) $P_6 = 220$ mice

$P_7 = 252$ mice

7 weeks

3 (a) $M_{10} = 25e^{-0.018 \times 10}$

$= 20.881...$

$= 20.9$ g

(b) $M_{20} = 25e^{-0.018 \times 20}$

$= 17.44...$

$= 17.4$ g

(c) $M_{50} = 25e^{-0.018 \times 50}$

$= 10.164...$

$= 10.2$ g

4 (a) $I(10) = 1000e^{0.05 \times 10}$

$= 1648.72...$

$= £1648.72$

(b) $I(13) = 1000e^{0.05 \times 13}$

$= £1915.54$

$I(14) = 1000e^{0.05 \times 14}$

$= £2013.75$

$\therefore$ investment doubles $13 < t < 14$

5 (a) $B(8) = 500e^{0.14 \times 8}$

$= 1532.42...$

$= 1532$ bacteria

(b) $B(9) = 1762.7$

$B(10) = 2027.6$

After 10 hours

1A

(6) $B(4) = 2000e^{-0.21\times4}$

$= 863$ bacteria

$B(10) = 2000e^{-0.21\times10}$

$= 244.9$

$= 245$ bacteria

(7) time $= 0$

(a) $D(0) = 25e^{-0.04\times0}$

$= 25$ units

(b) $D(6) = 25e^{-0.04\times6}$

$= 19.66...$

$= 19.7$ units (3 s.f.)

Higher Maths 1B LxL

① (a) $y = 5^3$

$\log_5 y = 3$

(b) $p = 4^t$

$\log_4 p = t$

(c) $f = g^h$

$\log_g f = h$

(d) $128 = 2^7$

$\log_2 128 = 7$

(e) ~~3x=~~ $y = e^x$

$\log_e y = x$

2(a) $3 = \log_2 8$

$2^3 = 8$

(b) $5 = \log_3 243$

$3^5 = 243$

(c) $y = \log_5 4$

$5^y = 4$

(d) $x = \log_m t$

$m^x = t$

(e) $3 = \log_4 y$

$4^3 = y$

③ (a) $4 = \log_2 a$

$2^4 = a$

$\underline{a = 16}$

(b) $3 = \log_9 a$

$9^3 = a$

$\underline{a = 729}$

(c) $5 = \log_{10} a$

$10^5 = a$

$\underline{a = 100\,000}$

(d) $a = \log_4 16$

$4^a = 16$

$\underline{a = 2}$

1C

1(a) $\log_7 3 + \log_7 9$

$= \log_7 (3 \times 9)$

$= \log_7 27$

(b) $\log_5 100 - \log_5 4$

$= \log_5 \left(\frac{100}{4}\right)$

$= \log_5 25$

$= \log_5 5^2$

$= 2 \cancel{\log_5 5}^{1}$

$= 2$

(c) $\log_4 12 - \log_4 3$

$= \log_4 \left(\frac{12}{3}\right)$

$= \log_4 4$

$= 1$

(d) $\log_{12} 36 + \log_{12} 4$

$= \log_{12} 144$

$= \log_{12} 12^2$

$= 2 \cancel{\log_{12} 12}^{1}$

$= 2$

(e) $\log_6 72 + \log_6 2 - \log_6 4$

$= \log_6 \left(\frac{72 \times 2}{4}\right)$

$= \log_6 36$

$= \log_6 6^2$

$= 2$

(f) $\log_9 24 - \log_9 8 - \log_9 21 + \log_9 7$

$= \log_9 \left(\frac{\cancel{24}^{3} \times \cancel{7}^{1}}{{}^{3}\cancel{21} \times \cancel{8}^{1}}\right)$

$= \log_9 (1)$

$= 0$

(g) $\log_4 12 + \log_4 8 - \log_4 3 - \log_4 2$

$= \log_4 \left(\frac{\cancel{12}^{4} \times \cancel{8}^{4}}{\cancel{3} \times \cancel{2}^{1}}\right)$

$= \log_4 16$

$= \log_4 4^2$

$= 2 \cancel{\log_4 4}^{1}$

$= 2$

(h) $\log_a \left(\frac{\cancel{5}^{1} \times \cancel{20}^{4}}{\cancel{5} \times \cancel{12}^{4}}\right)$

$= \log_a 1$

$= 0$

② (a) $\frac{1}{3} \log_3 27$

$= \log_3 27^{1/3}$

$= \log_3 \sqrt[3]{27}$

$= \log_3 3$

$= 1$

(b) $\log_2 \frac{1}{4}$

$= \log_2 \frac{1}{2^2}$

$= \log_2 2^{-2}$

$= -2 \log_2 2$

$= \underline{\underline{-2}}$

(c) $\log_p p^3$

$= 3 \cancel{\log_p p}^{1}$

$= 3$

(d) $\log_4 8^0$

$= 0 \log_4 8$

$= \underline{\underline{0}}$

(e) $4 \log_{16} 2$

$= \log_{16} 2^4$

$= \log_{16} 16$

$= 1$

③ (a) $\log_2 8 + 3\log_2 4 - \log_2 16$

$= \cancel{\log_2 8} \cancel{+ to}$

$= \log_2 2^3 + 3\log_2 2^2 - \log_2 2^4$

$= 3 \cancel{\log_2 2}^{1} + 6 \cancel{\log_2 2}^{1} - 4 \cancel{\log_2 2}^{1}$

$= \underline{\underline{5}}$

(b) $3 \log_{10} 5 + \log_{10} 8$

$= \log_{10} 5^3 + \log_{10} 8$

$= \log_{10} (125 \times 8)$

$= \log_{10} 1000$

$= \log_{10} 10^3$

$= 3 \log_{10} 10$

$= \underline{\underline{3}}$

3(c) $\log_4 36 - 2\log_4 12$

$= \log_4\left(\frac{\cancel{36}^{\,3}}{{}^{1}\cancel{12}\times 12}\right)$

$= \log_4 \frac{1}{4}$

$= \log_4 4^{-1}$

$= -\cancel{\log_4 4}^{\,1}$

$= -1$

(d) $2\log_3 9 + \log_3 5 - \log_3 15$

$= \log_3\left(\frac{9^2 \times \cancel{5}^{\,1}}{\cancel{15}\,3}\right)$

$= \log_3 27$

$= \log_3 3^3$

$= 3\cancel{\log_3 3}^{\,1}$

$= 3$

(e) $\log_5 \frac{1}{24} - \log_5 \frac{1}{8} + \log_5 15$

$= \log_5\left(\frac{\frac{1}{24}\times 15}{\frac{1}{8}}\right)$

$= \log_5\left(\frac{15}{\cancel{24}\,3} \times \frac{\cancel{8}^{\,1}}{1}\right)$

$= \log_5 5$

$= 1$

(f) $3\log_2 \frac{1}{4} + 2\log_2 16$

$= 3\log_2 \frac{1}{2^2} + 2\log_2 2^4$

$= 3\log_2 2^{-2} + 2\log_2 2^4$

$= -6\cancel{\log_2 2}^{\,1} + 8\cancel{\log_2 2}^{\,1}$

$= 2$

(g) $2\log_3 9 - 3\log_2 8$

$= 2\log_3 3^2 - 3\log_2 2^3$

$= 4\cancel{\log_3 3}^{\,1} - 9\cancel{\log_2 2}^{\,1}$

$= -5$

3(h) $\log_4 8 + \log_4 2 - 3\log_7 7$

$= \log_4(8\times 2) - 3\cancel{\log_7 7}$

$= \log_4 16 - 3$

$= \log_4 4^2 - 3$

$= 2\cancel{\log_4 4} - 3$

$= \underline{\underline{-1}}$

(i) $\log_5 25 + 3\log_2 \sqrt[3]{2}$

$= \log_5 5^2 + \log_2 \left(\sqrt[3]{2}\right)^3$

$= 2\cancel{\log_5 5} + \cancel{\log_2 2}$

$= 2 + 1$

$= \underline{\underline{3}}$

(j) $\log_6 18 - \log_6 3 + \log_3 \sqrt{3}$

$= \log_6\left(\frac{18}{3}\right) + \log_3 (3)^{\frac{1}{2}}$

$= \cancel{\log_6 6} + \frac{1}{2}\cancel{\log_3 3}$

$= \underline{\underline{1\tfrac{1}{2}}}$

(k) $2\log_4 16 + 3\log_2 8 - \log_2 \sqrt{8}$

$= 2\log_4 4^2 + 3\log_2 2^3 - \log_2 \sqrt{2^3}$

$= 4\cancel{\log_4 4} + 9\cancel{\log_2 2} - \log_2 (2)^{3/2}$

$= 13 - \frac{3}{2}$

$= \underline{\underline{11\tfrac{1}{2}}}$

4) $A = -\log_{10} 8 + \log_{10} 80$

$A = \log_{10}\left(\frac{80}{8}\right)$

$A = \log_{10} 10$

$\underline{\underline{A = 1}}$

5) $L = 10\log_{10} P_1^2 - 20\log_{10} P_0$

$= 20\log_{10} P_1 - 20\log_{10} P_0$

$= 20\log_{10} 2000 - 20\log_{10} 2\times 10^{-5}$

$= 20\left(\log_{10}\left(\frac{2000}{2\times 10^{-5}}\right)\right)$

$= 20\log_{10}\left(\frac{10^3}{10^{-5}}\right)$

$= 20\log_{10} 10^8$

$= 160\log_{10} 10$

$= \underline{\underline{160\ dB}}$

Higher Maths LoL

1D

1(a) $y = \log_e 3$

$e^y = 3$

(b) $4 = \log_e x$

$x = e^4$

(c) $p = \log_e q$

$q = e^p$

(d) $y = \log_{10} 5$

$10^y = 5$

(e) $3 = \log_{10} x$

$x = 10^3$

2(a) $y = e^5$

$\log_e y = 5$

(b) $2 = e^x$

$\log_e 2 = x$

(c) $f = e^x$

$x = \log_e f$

(d) $y = 10^x$

$\log_{10} y = x$

(e) $x = 10^y$

$\log_{10} x = y$

3(a) $\log_e 8$

$= 2.0794\ldots$

$= \underline{2.08}$

(b) $\log_{10} 25$

$= \underline{1.40}$

(c) $e^{0.8}$

$= \underline{2.23}$

(d) $10^{-0.2}$

$= \underline{0.63}$

4(a) $\log_e e^2$

$= 2 \cancel{\log_e e}^{1}$

$= \underline{2}$

1D

4(b) $\log_e \frac{1}{e}$

$= \log_e e^{-1}$

$= -\cancel{\log_e e}^{1}$

$= \underline{\underline{-1}}$

4(c) $6 \log_e \sqrt[3]{e}$

$= 6 \log_e e^{1/3}$

$= \frac{1}{3} \times 6 \cancel{\log_e e}^{1}$

$= \underline{\underline{2}}$

4(d) $\log_e e^3 + 2 \log_e \sqrt[3]{e} - 4 \log_e \sqrt{e}$

$= \log_e e^3 + 2 \log_e e^{1/3} - 4 \log_e e^{1/2}$

$= 3 \cancel{\log_e e}^{1} + \frac{1}{3} \times 2 \cancel{\log_e e}^{1} - \frac{1}{2} \times 4 \cancel{\log_e e}^{1}$

$= 3 + \frac{2}{3} - 2$

$= \underline{\underline{\frac{5}{3}}}$

5(a) $\frac{\log_3 4p^2}{\log_3 2p}$

$= \frac{\log_3 (2p)^2}{\log_3 (2p)}$

$= \frac{2 \cancel{\log_3 2p}^{1}}{\cancel{\log_3 2p}^{1}}$

$= \underline{\underline{2}}$

(b) $\frac{\log_k (8t^3)}{\log_k (2t)}$

$= \frac{\log_k (2t)^3}{\log_k (2t)}$

$= \frac{3 \cancel{\log_k (2t)}^{1}}{\cancel{\log_k 2t}^{1}}$

$= \underline{\underline{3}}$

(c) $\frac{\log_a (27q^3)}{\log_a (9q^2)}$

$= \frac{\log_a (3q)^3}{\log_a (3q)^2}$

$= \frac{3 \cancel{\log_a (2q)}}{2 \cancel{\log_a (2q)}}$

$= \underline{\underline{\frac{3}{2}}}$

Higher Maths L&L

1E

1(a) $\log_3 x = 5$

$x = 3^5$

$\underline{x = 243}$

(c) $\log_5 x = 3$

$x = 5^3$

$\underline{x = 125}$

(e) $3\log_e x = 12$

$\log_e x = 4$

$x = e^4$

$x = 54.6$ (3 s.f.)

2(a) $\log_2 8x = 5$

$8x = 2^5$

$8x = 32$

$\underline{x = 4}$

(c) $\log_3 \frac{x}{4} = 5$

$\frac{x}{4} = 5^3$

$x/4 = 125$

$\underline{x = 500}$

(e) $3\log_6 2x = 12$

$\log_6 2x = 4$

$2x = 6^4$

$2x = 1296$

$x = 648$

(g) $\log_4 (x-1) = 3$

$(x-1) = 4^3$

$x-1 = 64$

$\underline{x = 65}$

3(a) $10^x = 3$

$x = \log_{10} 3$

$x = 0.477$ (3 s.f.)

(c) $4^x = 262144$

$x = \log_4 262144$

$\underline{x = 9}$

4(a) $4e^x = 16$

$e^x = 4$

$x = \log_e 4$

$\underline{x = 1.39}$ 3(s.f.)

4 (c) $\frac{3^x}{2} = 40.5$

$3^x = 81$

$x = \log_3 81$

$x = \log_3 3^4$

$x = 4\cancelto{1}{\log_3 3}$

$\underline{\underline{x = 4}}$

(e) $2^{3x} = 512$

$3x = \log_2 512$

$3x = 9$

$\underline{\underline{x = 3}}$

(g) $3^{\frac{2x}{5}} = 81$

$\frac{2x}{5} = \log_3 81$

$\frac{2x}{5} = \log_3 3^4$

$\frac{2x}{5} = 4\cancelto{1}{\log_3 3}$

$2x = 4 \times 5$

$\underline{\underline{x = 10}}$

5 (a) $\log_x 25 = 2$

$25 = x^2$

$\underline{\underline{x = 5}}$

(c) $\log_x 6561 = 4$

$x^4 = 6561$

$x = \sqrt[4]{6561}$

$\underline{\underline{x = 9}}$

(e) $\log_{x-1} 2401 = 4$

$(x-1)^4 = 2401$

$(x-1) = \sqrt[4]{2401}$

$x - 1 = 7$

$\underline{\underline{x = 8}}$

1E Higher Maths L & L

1(a) $\log_a x - \log_a 7 = \log_a 4$

$\cancel{\log_a}\left(\frac{x}{7}\right) = \cancel{\log_a} 4$

$x = 28$

(c) $\log_k 3 + \log_k x = \log_k 7$

$\cancel{\log_k} 3x = \cancel{\log_k} 7$

$x = \frac{7}{3}$

(e) $\log_u x - \log_u 4 + \log_u 12 = \log_u 2$

$\cancel{\log_u}\left(\frac{12x}{4}\right) = \cancel{\log_u}(2)$

$3x = 2$

$x = \frac{2}{3}$

(g) $\log_b(18) - \log_b(x+1) = \log_b(6)$

$\cancel{\log_b}\left(\frac{18}{x+1}\right) = \cancel{\log_b} 6$

$\frac{18}{x+1} = 6$

$18 = 6(x+1)$

$18 = 6x + 6$

$12 = 6x$

$x = 2$

2(a) $\log_a 4x + \log_a(x+1) = \log_a 8$

$\cancel{\log_a}(4x(x+1)) = \cancel{\log_a} 8$

$4x^2 + 4x = 8$

$4x^2 + 4x - 8 = 0$

$4(x^2 + x - 2) = 0$

$4(x+2)(x-1) = 0$

$\cancel{x = -2}$, $x = 1$ as $x > 0$

(c) $\log_{10}(x-2) + \log_{10}(x-3) = \log_{10} 2$

$\cancel{\log_{10}}[(x-2)(x-3)] = \cancel{\log_{10}} 2$

$x^2 - 5x + 6 = 2$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1$, $x = 4$

(e) $\log_q(x^2-4) - \log_q(x+2) = \log_q 7$

$\cancel{\log_q}\left(\frac{x^2-4}{x+2}\right) = \cancel{\log_q} 7$

$x^2 - 4 = 7(x+2)$

$x^2 - 4 = 7x + 14$

$x^2 - 7x - 18 = 0$

$(x-9)(x+2) = 0$

$x = 9$, $\cancel{x = -2}$ as $x > 0$

2(g) $\log_5 x + \log_5(x+1) + \log_5(x-1) = \log_5 6$

$\cancel{\log_5}\left[x(x-1)(x+1)\right] = \cancel{\log_5} 6 \quad \rightarrow \quad (x-1)\,x\,(x+1) = 6$

$x(x^2-1) = 6$

$x^3 - x - 6 = 0$

3 consecutive numbers multiplied together $= 6 \therefore \underline{x = 2}$

↳ we can solve this later in the course but not now.

(3) $\left.\begin{array}{l} t = \log_2 a \\ t = \frac{2}{3}\log_2 b \end{array}\right\}$ substitute into x and y.

$\log_2 a = \frac{2}{3}\log_2 b$

$\cancel{\log_2} a = \cancel{\log_2} b^{2/3}$

$a = \sqrt[3]{b^2}$

$b = 8 \Rightarrow a = \sqrt[3]{8^2}$

$a = 2^2$

$\underline{\underline{a = 4}}$

Higher Maths L&L

1G

1(a) $\log_4 x + \log_4 8 = 2$

$\log_4 8x = 2$

$8x = 4^2$

$8x = 16$

$\underline{\underline{x = 2}}$

(b) $\log_3 x + \log_2 8 = 4$

note: bases are different so we can not add logs.

$\log_3 x + \log_2 2^3 = 4$

$\log_3 x + 3\cancel{\log_2 2}^{1} = 4$

$\log_3 x + 3 = 4$

$\log_3 x = 1$

$\underline{\underline{x = 3}}$

(e) $2\log_5 x - 3\log_4 8 = 1$

$\log_5 x^2 - \log_4 8^3 = 1$

$\log_5 x^2 - \log_4 64 = 1$

(e) continued

$\log_5 x^2 - \log_4 4^3 = 1$

$\log_5 x^2 - 3\cancel{\log_4 4}^{1} = 1$

$\log_5 x^2 = 4$

$x^2 = 5^4$

$x = 5^2$

$\underline{\underline{x = 25}}$

2(a) $\log_4 x + \log_4 (x+12) = 3$

$\log_4 [x(x+12)] = 3$

$x(x+12) = 4^3$

$x^2 + 12x - 64 = 0$

$(x+16)(x-4) = 0$

$\cancel{x = -16}$, $\underline{\underline{x = 4}}$ as $x > 0$

(c) $\log_2 (2x-1) + \log_2 (2x+1) = 3$

$\log_2 [(2x-1)(2x+1)] = 3$

$(2x-1)(2x+1) = 2^3$

$4x^2 - 1 = 8$

$4x^2 - 9 = 0$

$(2x+3)(2x-3) = 0$

$\cancel{x = -\frac{3}{2}}$ $\underline{\underline{x = \frac{3}{2}}}$ as $x > 0$

16

2(e) $2\log_2(x+1) - \log_2(2x) = 1$

$\log_2 \frac{(x+1)^2}{2x} = 1$

$\frac{(x+1)^2}{2x} = 2^1$

$x^2+2x+1 = 2(2x)$

$x^2+2x+1 = 4x$

$x^2-2x+1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

Higher Maths L&L

1H

(1)(a) $I_0 = 7$
$t = 7$

$I_7 = 7e^{0.43 \times 7}$

$= 142$ people

(b) $700 = 7e^{0.43t}$

$100 = e^{0.43t}$

~~$\log_e 100 = \log_e 0.4$~~

$\log_e 100 = \log_e e^{0.43t}$

$\log_e 100 = 0.43t \log_e e$

$t = \dfrac{\log_e(100)}{0.43}$

$= 10.7$ days

after 11 days 100+ people will be infected.

(2)(a) V_n = value after n years

$V_n = 20000(1.024)^n$

$V_5 = 20000(1.024)^5$

$= £22518.00$

(b) $25000 = 20000(1.024)^n$

$\dfrac{25000}{20000} = (1.024)^n$

$\log_e\left(\dfrac{25000}{20000}\right) = \log_e(1.024)^n$

$\log_e\left(\dfrac{25000}{20000}\right) = n\log_e(1.024)$

$n = \dfrac{\log_e\left(\dfrac{25000}{20000}\right)}{\log_e(1.024)}$

$n = 9.4$ years

3 (a) $t = 0$ at start

$$B_0 = 200e^{0.8 \times 0}$$

$$\underline{B_0 = 200}$$

(b) $600 = 200e^{0.8t}$

$$3 = e^{0.8t}$$

$$\log_e 3 = \log_e e^{0.8t}$$

$$\log_e 3 = 0.8t \log_e e$$

$$\frac{\log_e(3)}{0.8} = t$$

$t = 1.373\ldots$ hours

$t = 1.373 \times 60$ mins

$t = 82.39$ mins

$t = 1$ hour 23 minute

4 (a) $\cancel{50 = 100e^{-kt}}$

$$N_0 = 100$$

$$N_{5.27} = 50$$

$$t = 5.27$$

$$50 = 100e^{-5.27k}$$

$$\frac{1}{2} = e^{-5.27k}$$

$$\log_e \frac{1}{2} = \log_e e^{-5.27k}$$

$$\log_e \frac{1}{2} = -5.27k$$

$$\frac{\log_e \frac{1}{2}}{-5.27} = k$$

$$\underline{k = 0.132 \text{ (3 S.F.)}}$$

(b) 30% of original $= 0.3N_0$

$$0.3\cancel{N_0} = \cancel{N_0}e^{-0.132t}$$

(using value of k from part 1)

$$\log_e 0.3 = \log_e e^{-0.132t}$$

$$\log_e 0.3 = -0.132t$$

$$\frac{\log_e 0.3}{-0.132} = t$$

$$\underline{t = 9.12 \text{ years}}$$

④(c) $t = 12$

$$N_{12} = N_0 e^{-0.132 \times 12}$$

$$N_{12} = 0.205 N_0$$

20.5% of original mass (N_0)

⑤(a) $M_t = M_0 e^{-kt}$

$M_0 = 100$

$t = 10$

$M_{10} = 95.1$

$$95.1 = 100 e^{-10k}$$

$$\frac{95.1}{100} = e^{-10k}$$

$$\log_e\left(\frac{95.1}{100}\right) = -10k$$

$$\frac{\log_e\left(\frac{95.1}{100}\right)}{-10} = k$$

$$k = 5.02 \times 10^{-3}$$

$$k = \cancel{0.000502}\ 0.00502$$

(b) Half life, t, $N_t = \frac{1}{2} N_0$.

$$\frac{1}{2} N_0 = N_0 e^{-0.00502 t}$$

$\frac{1}{2}$ original mass

k from part (a)

$$\frac{1}{2} = e^{-0.00502 t}$$

$$\log_e \frac{1}{2} = -0.00502 t$$

$$\frac{\log_e \frac{1}{2}}{-0.00502} = t$$

$t = 138.1$ years

Higher Maths Lab

1 I

1 (a) $pH = -\frac{1}{2}\log_{10}(1.8\times10^{-5}) + \frac{1}{2}\log_{10}(2)$ evaluate on calculator.

$\underline{pH = 2.52}$

(b) $-\log_{10} H = -\frac{1}{2}\log_{10} Ka + \frac{1}{2}\log_{10} c$

$-2\log_{10} H = -\log_{10} Ka + \log_{10} c$ • double both sides to remove fraction

$\log_{10} Ka - 2\log_{10} H = \log_{10} c$

$\log_{10} Ka - \log_{10} H^2 = \log_{10} c$

$\log_{10}\left(\frac{Ka}{H^2}\right) = \log_{10} c$

$\underline{\log_{10} c = \log_{10}\left(\frac{Ka}{H^2}\right)}$

1I

(2a) $I_1 = 10\,000 W$

$I_2 = 0.1 W$

$$D = 10 \log_{10}\left(\frac{10\,000}{0.1}\right)$$

$$D = 10 \log_{10}(100\,000)$$

$$D = 10 \log_{10}(10)^5$$

$$D = 5 \times 10 \cancel{\log_{10} 10}^{1}$$

$$\underline{D = 50 \text{ dB}}$$

(4)(a) Particle 1 $\phi = 2$

$$2 = -\log_2 \frac{D_1}{D_0}$$

Particle 2 ($D = 4D_1$) $\phi = -\log_2 \frac{4D_1}{D_0}$

$$\phi = -\log_2 4 - \log_2 \frac{D_1}{D_0}$$

$$\phi = -\log_2 2^2 + 2 \leftarrow$$ • from above $2 = -\log_2 \frac{D_1}{D_0}$

$$\phi = -2 \cancel{\log_2 2}^{1} + 2$$

$$\underline{\phi = 0}$$

Higher Maths L&L

IJ

(1) $y = kx^n$

$\log_{10} y = \log_{10} kx^n$

$\log_{10} y = \log_{10} k + \log_{10} x^n$

$\log_{10} y = \log_{10} k + n \log_{10} x$

$\log_{10} y = n \log_{10} x + \log_{10} k$

$Y = m X + c$

straight line with gradient n & y-intercept $\log_{10} k$

From graph $m = \frac{5.5 - 0.7}{0.8 - 0}$

$= \frac{4.8}{0.8}$

$= \underline{\underline{6}}$ $\underline{n = 6}$

$\log_{10} k = 0.7$

$k = 10^{0.7}$

$\underline{k \approx 5}$

$\underline{\underline{y = 5x^6}}$

1(b) $y = Kx^n$

$\log_e y = \log_e Kx^n$

$\log_e y = \log_e x^n + \log_e K$

$\log_e y = n \log_e x + \log_e K$

$Y = mX + c$

straight line with gradient n, y-intercept $\log_e K$

from graph $m = \dfrac{12.69 - 0.69}{2 - 0}$

$m = \underline{\underline{6}}$

$\Rightarrow n = 6$

$\log_e K = 0.69$

$K = e^{0.69}$

$K \approx 2.0$

$\underline{\underline{y = 2x^6}}$

① (c) see 1(a)

$$m = \frac{2.2 - 1.3}{0.4 - 0.1}$$

$$m = \frac{0.9}{0.3}$$

$m = 3$

$\Rightarrow n = 3$

$y - b = m(x - a)$

$y - 1.3 = 3(x - 0.1)$

$y - 1.3 = 3x - 0.3$

$y = 3x + \underline{1}$ y-int $= 1$

$\log_{10} K = 1$

$K = 10^1$

$\underline{K = 10}$

$y = Kx^n$

$\underline{y = 10x^3}$

2 (a) $y = ab^x$

$\log_{10} y = \log_{10} ab^x$

$\log_{10} y = \log_{10} b^x + \log_{10} a$

$\log_{10} y = x \log_{10} b + \log_{10} a$

$Y = mX + c$

$m = \log_{10} b \quad c = \log_{10} a$

Straight line with gradient $\log_{10} b$ & y-intercept $\log_{10} a$

$m = \dfrac{1.08 - 0.90}{0.6 - 0}$

$m = \dfrac{0.18}{0.6}$

$m = 3/10$

$\log_{10} b = 3/10$

$b = 10^{3/10}$

$\underline{\underline{b \approx 2}}$

y-intercept

$\log_{10} a = 0.9$

$a = 10^{0.9}$

$\underline{\underline{a = 7.94}}$

$\underline{\underline{y = 7.94(2^x)}}$

(2) (b)

$$y = ab^x$$

$$\log_e y = \log_e (ab^x)$$

$$\log_e y = \log_e b^x + \log_e a$$

$$\log_e y = x \log_e b + \log_e a$$

$$Y = mX + c$$

straight line with gradient $= \log_e b$, y-int $= \log_e a$

$$m = \frac{5.5 - 2.2}{3 - 0}$$

$$= 1.1$$

$$\log_e b = 1.1$$

$$b = e^{1.1}$$

$$\underline{b \approx 3}$$

$$\log_e a = 2.2$$

$$a = e^{2.2}$$

$$\underline{a = 9.03}$$

$$\underline{y = 9.03(3^x)}$$

2(c) $y = ab^x$

$\log_e y = \log_e ab^x$

$\log_e y = \log_e b^x + \log_e a$

$\log_e y = x \log_e b + \log_e a$

$m = \log_e b \qquad c = \log_e a$

$$m = \frac{4.04 - 3.08}{1.4 - 0.8}$$

$m = 1.6$

$\log_e b = 1.6$

$b = e^{1.6}$

$\underline{\underline{b = 4.953}}$

$y - b = m(x - a)$

$y - 3.08 = 1.6(x - 0.8)$

$y - 3.08 = 1.6x - 1.28$

$y = 1.6x + \underline{\underline{1.8}}$

$\log_e a = 1.8$

$a = e^{1.8}$

$a = 6.050$

$\underline{\underline{y = 6.05(4.953^x)}}$

③ (a) $y = kx^n$ ← check to see if this matches (a) or (b).

$\log y = \log kx^n$

$\log y = \log x^n + \log k$

$$\log y = \underset{(m)}{n \log x} + \underset{(c)}{\log k}$$

(b) log v log graph so matches graph (b)

$\log_{10} y = n \log_{10} x + \log_{10} k$

$$m = \frac{3 - 1.4}{0 - 0.8}$$

$m = -2$

$\Rightarrow \underline{\underline{n = -2}}$

$\log_{10} k = 3$ (y-intercept)

$k = 10^3$

$\underline{\underline{k = 1000}}$

$$\underline{\underline{y = 1000x^{-2}}}$$

3 (a) $y = ab^x$

* note 3b on previous page.

$\log_e y = \log_e ab^x$

$\log_e y = \log_e b^x + \log_e a$

$\log_e y = x \log_e b + \log_e a$

$m = \log_e b \qquad c = \log_e a$

$m = \frac{1-0}{0-0.5}$

$= -2$

$\log_e b = -2$

$\underline{\underline{b = e^{-2}}}$

$\log_e a = 1$

$a = e^1$

$\underline{\underline{a = e}}$

$y = e \cdot (e^{-2})^x$

$y = e \cdot e^{-2x}$

$y = \underline{\underline{e^{1-2x}}}$